Probability & Statistics (1)

Axioms of Probability (I)

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Outlines

1. Introduction
2. Sample Space and Events
3. Axioms of Probability
4. Some Simple Propositions
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Introduction

- 在介紹完排列組合(permutations and combinations)之後，接下來我開始正式進入到機率(probability)。這個章節主要介紹機率的重要公理(axiom)，同時還有一些從公理延伸出來的重要命題(proposition)。
- 我們會先從樣本空間(sample space)開始介紹，再提到集合(set)的概念，才會帶到機率公理與命題。

Sample Space and Events

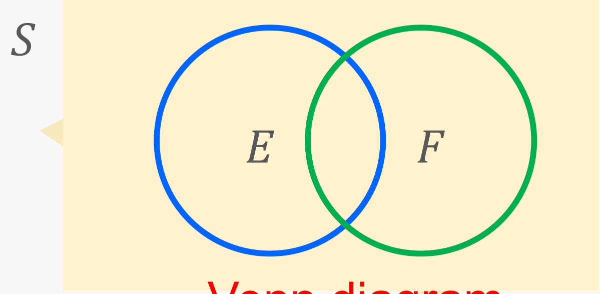
- **樣本空間(sample space)**

Definition from Sheldon Ross's Textbook

> This set of all possible outcomes of an experiment is known as the ***sample space*** of the experiment and is denoted by ***S***.

所謂樣本空間就是將所有可能出現實驗結果的集合稱之。

Sample Space and Events



Venn diagram

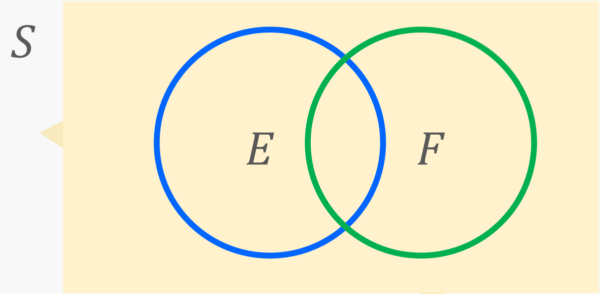
- **樣本空間(sample space)**

以萬年常用的硬幣為例，分為人頭(head；H)與背面(tail；T)。假設我們投擲兩次硬幣，所有的可能性就會有$2 \times 2 = 4$種，包含: 兩次人頭、兩次背面、一次人頭一次背面、與一次背面一次人頭。

這個可能性的集合就是擲兩次硬幣的樣本空間，我們可以用數學集合的方式來呈現:

$$S = \{(H,H),(T,T),(H,T),(T,H)\}$$

Sample Space and Events

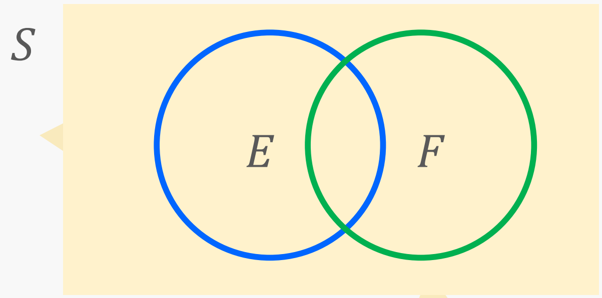


- **樣本空間(sample space)**

同樣的道理，當我們今天改成投三個骰子的時候，我們可以得到其樣本空間中的樣本數應為$6 \times 6 \times 6 = 216$種組合。然而這樣全部列出來實在是太龐大，再加上我們已知每次擲骰子出來的樣本空間都只有1到6這六個數字的可能性，因此我們就可以將投擲三個骰子的樣本空間表示為:

$$S = \{(i, j, k): i, j, k = 1,2,3,4,5,6\}$$

Sample Space and Events



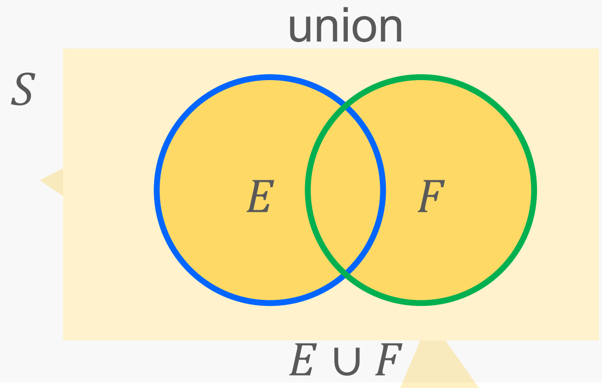
- 事件(event)

所有在樣本空間內的子集合(subset)都成為一個事件(event；E)；換句話說，一個事件可以視為該實驗的部分結果的子集合。

從前面投擲兩次硬幣的集合中，當我們只看第一次為人頭的事件，此時我們可以寫成

$$E = \{(H, H), (H, T)\}$$

Sample Space and Events



- **事件(event)**

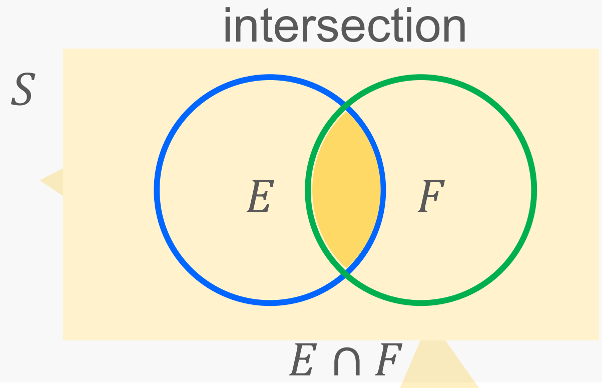
所以當我們今天有兩個事件，分別為E與F，其子集合為:

$$E = \{(H,H),(H,T)\}$$
$$F = \{(T,T)\}$$

此時， E與F事件的**聯集(union)**，以數學符號表示為$\boldsymbol{E \cup F}$，而其聯集的集合為:

$$E \cup F = \{(H,H),(H,T),(T,T)\}$$

Sample Space and Events



- **事件(event)**

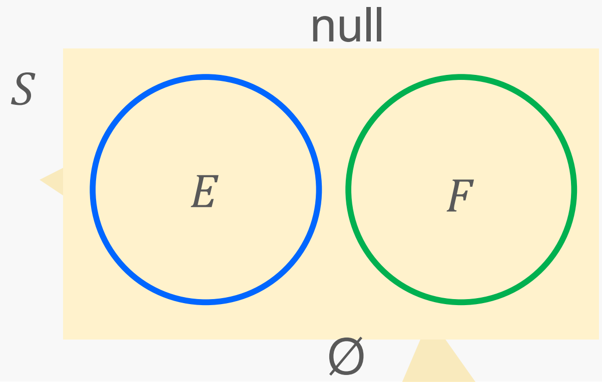
如果說，有E與F兩個事件，其子集合分別為:

$$E = \{(H,H),(H,T),(T,H)\}$$
$$F = \{(H,H),(H,T),(T,T)\}$$

則E與F兩個事件的**交集(intersection)**，數學符號表示為$\boldsymbol{E \cap F}$或是$\boldsymbol{EF}$，可以被定義為:

$$E \cap F = \{(H,H),(H,T)\}$$

Sample Space and Events



- **事件(event)**

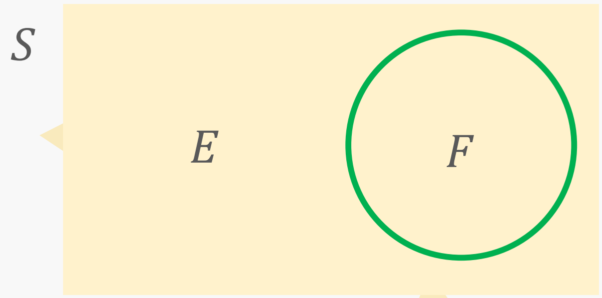
如果說，有E與F兩個事件，其子集合分別為:

$$E = \{(H,H),(H,T)\}$$
$$F = \{(T,T),(T,H)\}$$

則E與F兩個事件的交集(intersection)就會為空集合(null event ; denote it by Ø)，所以我們可以稱兩個事件$\boldsymbol{E \cap F}$為**互斥(mutually exclusive)**。

Sample Space and Events

contained



$E \supset F \text{ or } F \subset E$

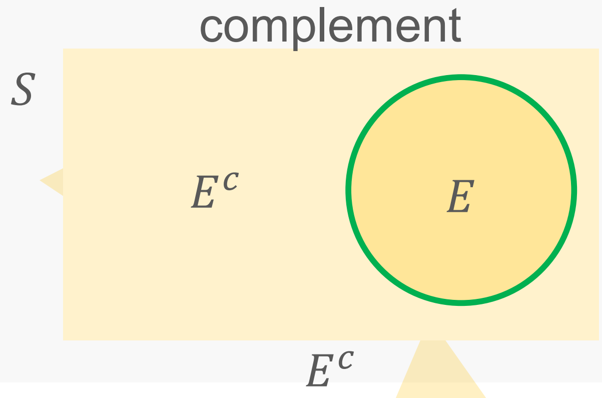
- 事件(event)

除了上述常見的集合關係以外，當今天事件E完全涵蓋事件F的時候，我們就可以說是件E**包含(contained)**事件F；換句話說，事件F為事件E的**子集合(subset)**，亦或是事件E為事件F的**母集合(superset)**。

在數學符號的表示式，可以使用$\boldsymbol{E \supset F}$或是$\boldsymbol{F \subset E}$。

如果今天$E \supset F = F \supset E$同時發生，我們就會知道事件E等於事件F，也可以用$E = F$來表示。

Sample Space and Events



- **事件(event)**

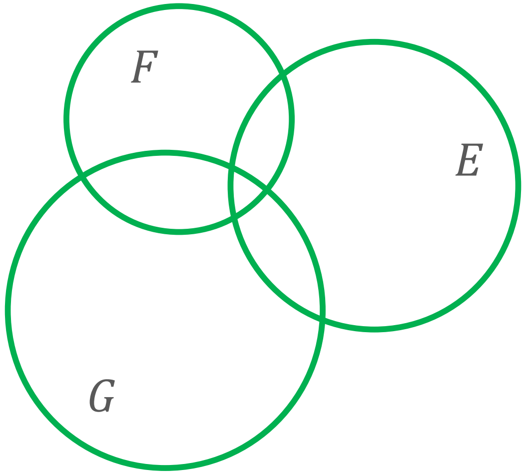
最後一個是**差集(相對差集)[relative complement]**與**補集(絕對差集) [absolute complement]** 。這兩者之間最大的差異就在於兩個集合是否有相互包含，如果倆著完全沒有重疊，則稱為絕對差集(補集)；反之則稱為相對差集(差集)。在我們這堂課程中，都只會使用到補集(絕對差集)的觀念。

在數學的表示中，我們會使用E^c(or E')來代表事件E的補集。

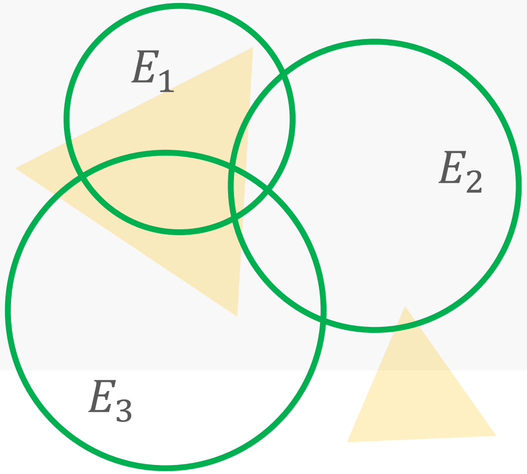
Sample Space and Events

- 事件與集合的規則

交換律 (Commutative laws)	$E \cup F = F \cup E$	$EF = FE$
結合律 (Associative laws)	$(E \cup F) \cup G = E \cup (F \cup G)$	$(EF)G = E(FG)$
分配律 (Distributive laws)	$(E \cup F)G = EG \cup FG$	$EF \cup G = (E \cup G)(F \cup G)$



Sample Space and Events



- **DeMorgan's laws**

First law

$$\left(\bigcup_{i=1}^{n} E_i\right)^c = \bigcap_{i=1}^{n} E_i^c$$

Second law

$$\left(\bigcap_{i=1}^{n} E_i\right)^c = \bigcup_{i=1}^{n} E_i^c$$

From the first law to the second law, …

$$\left(\bigcup_{i=1}^{n} E_i^c\right)^c = \bigcap_{i=1}^{n} (E_i^c)^c$$

Since $(E^c)^c = E$

$$\left(\bigcup_{i=1}^{n} E_i^c\right)^c = \bigcap_{i=1}^{n} E_i$$

Take complements of both sides,…

$$\bigcup_{i=1}^{n} E_i^c = \left(\bigcap_{i=1}^{n} E_i\right)^c$$

Axioms of Probability

- From Sheldon Ross's textbook, the probability of an event is in terms of its relative frequency.
- We suppose that an experiment, whose sample space is $\boldsymbol{S}$, is repeatedly performed under exactly the same conditions.
- For each event $\boldsymbol{E}$ of the sample space $\boldsymbol{S}$, we define $\boldsymbol{n}(\boldsymbol{E})$ to be the number of times in the first $\boldsymbol{n}$ repetitions of the experiment that the event $\boldsymbol{E}$ occurs. Then $\boldsymbol{P}(\boldsymbol{E})$, the probability of the event $\boldsymbol{E}$, is defined as …

$$P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$$

Axioms of Probability

- 這樣的formula其實存在著有許多的潛在的疑慮?
  - 我怎麼知道前面的發生頻率會與後者相同?
  - 或是發生頻率…
  - …
  - …
  - …
  - …

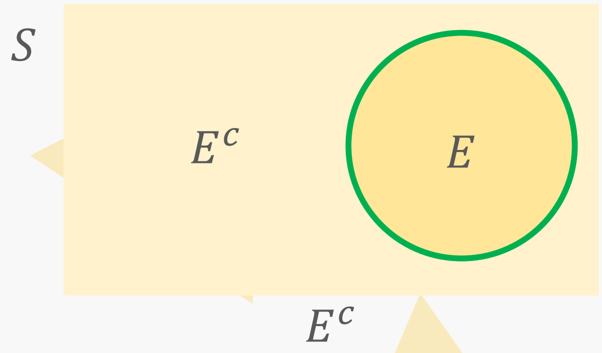
Discussion it online!

Axioms of Probability

- 機率公理(axioms of probability)

Axiom 1	$0 \le P(E) \le 1$
Axiom 2	$P(S) = 1$
Axiom 3	*For any sequence of mutually exclusive events* $E_1, E_2, \ldots (E_i E_j = \emptyset, where\ i \ne j)$ $P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

Axioms of Probability



- 如果我們有一連串的事件$E_1, E_2, \cdots, E_i$，當$E_1 = S$與$E_i = \emptyset$的前提下，所有事件彼此之間為互斥(mutually exclusive)，因為$S = \bigcup_{i=1}^{\infty} E_i$。根據機率第三公理，我們可以知道:

$$P(S) = \sum_{i=1}^{\infty} P(E_i) = P(S) + \sum_{i=2}^{\infty} P(\emptyset)$$

$$P(\emptyset) = 0$$

如果有一個有限序列的互斥事件$E_1, E_2, \cdots, E_n$，則我們可以得到:

$$P\left(\bigcup_{1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i)$$

Axioms of Probability

- **範例一**

今天某電器行辦抽獎活動，只要抽中紅色球的客人即可獲得免費訂閱報紙一年。在抽獎箱裡面，總共有十種顏色不同的球，假設每顆球被抽中的機率相同，那麼中獎機率為何?

呈上題，如果今天每顆球有編號1-10，抽中質數的人為中獎，請問中獎機率為何?

Some Simple Propositions

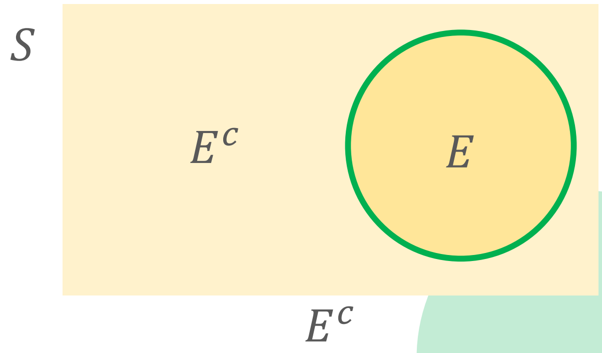
Axiom 1	$0 \le P(E) \le 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

- 從前面的定義中，我們已經了解到事件E的補集(E^c)與事件E(E)為互斥事件(mutually exclusive)，因此我們可以從事件E的補集與事件E的聯集($E \cup E^c = S$)獲得整個樣本空間S。
- 從第二與第三機率公理中，我們可以用數學的表示如下，

$$1 = P(S) = P(E \cup E^c) = P(E) + P(E^c)$$

- 獲得我們第一個命題(**Proposition 1**)

$$P(E^c) = 1 - P(E)$$



Some Simple Propositions

Axiom 1	$0 \le P(E) \le 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

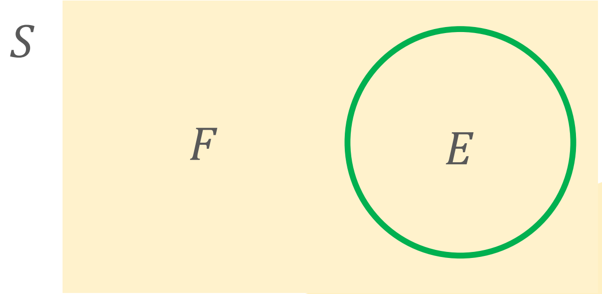
- **範例二**

假設我們玩抽撲克牌的遊戲，假設抽到每一張的機率皆相同，那麼抽一張抽到A的機率多少? 抽一張抽到不是人頭(J、Q、K)的機率有多少?

Some Simple Propositions

Axiom 1	$0 \le P(E) \le 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

- **Proposition 2**: If $E \subset F$, then $P(E) \le P(F)$.



$F \supset E$ or $E \subset F$

Proof. $\because E \subset F$，$\because F$可以被表示為

$$F = E \cup E^c F$$

因為$E \cup E^c F$為互斥事件，根據機率第三公理可以得知

$$P(F) = P(E) + P(E^c F)$$

又因為$P(E^c F) \ge 0$，所以$P(E) \le P(F)$。

Some Simple Propositions

Axiom 1	$0 \le P(E) \le 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

- **Proposition 3:** $P(E \cup F) = P(E) + P(F) - P(EF)$

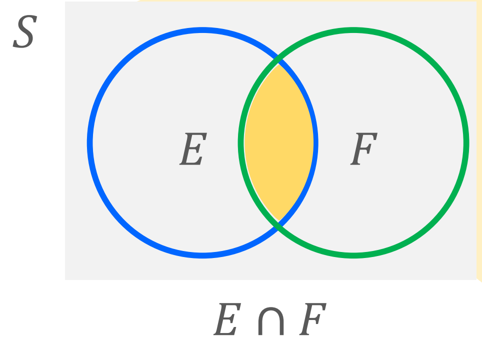
Proof. 依據機率第三定理，我們可以$E \cup F$拆成一組互斥的事件$E \cup E^c F$，因此我們可以得到:

$$P(E \cup F) = P(E \cup E^c F) = P(E) + P(E^c F)$$

由於，$F = EF \cup E^c F$，因此可以再次套入機率第三定理

$$P(F) = P(EF) + P(E^c F)$$

此時，可以將上述兩式合併得出，$P(E) + P(F) - P(EF)$

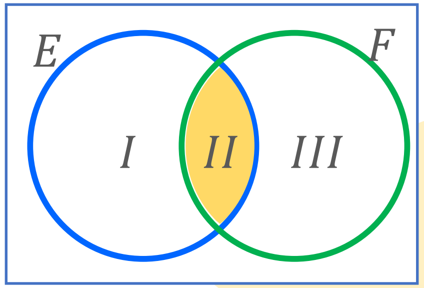


Some Simple Propositions

Axiom 1	$0 \le P(E) \le 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

- **Proposition 3:** $P(E \cup F) = P(E) + P(F) - P(EF)$

Proof.



$$E \cup F = I \cup II \cup III$$
$$E = I \cup II$$
$$F = II \cup III$$

子集合$I, II, and\ III$ 彼此為互斥事件，所以可以帶入機率第三公理

$$P(E \cup F) = P(I) + P(II) + P(III)$$
$$P(E) = P(I) + P(II)$$
$$P(F) = P(II) + P(III)$$

所以可以得出:$P(E \cup F) = P(E) + P(F) - P(II)$, 而$P(II) = P(EF)$。

Some Simple Propositions

Axiom 1	$0 \leq P(E) \leq 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

- **範例三**

今天是你是電商後台的資料分析人員，現在主管需要你去評估一個使用者介面的成效。根據過去一年來的數據顯示，人們會透過社群媒體廣告購買的機率為0.4，透過平面廣告購買的機率為0.2，需透過社群媒體廣告與平面廣告才會有購買行為的機率為0.15。

(1) 試問透過任一平台廣告而產生購買行為的機率為何?

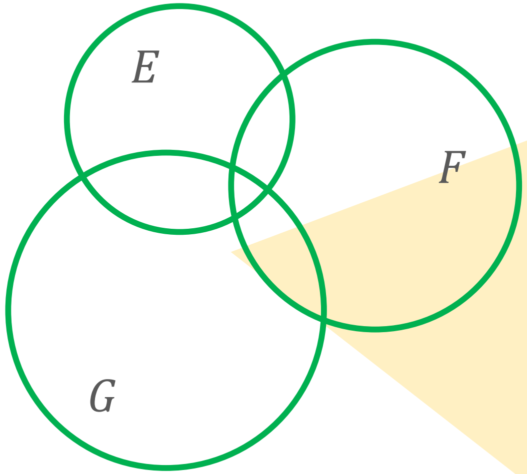
(2) 試問不透過社群媒體廣告或平面廣告所產生購買行為機率為何?

Some Simple Propositions

Axiom 1	$0 \le P(E) \le 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

- **範例四**

假設今天有三個彼此不互斥的事件E、F、G。試求只少其中一個事件發生的機率為何?



Solution:

$$P(E \cup F \cup G) = P[(E \cup F) \cup G]$$
$$= P(E \cup F) + P(G) - P[(E \cup F) \cap G]$$
$$= P(E) + P(F) - P(E \cap F) + P(G) - P([E \cap G] \cup [F \cap G])$$
$$= P(E) + P(F) - P(E \cap F) + P(G) - P(E \cap G) - P(F \cap G) + P(EG \cap FG)$$
$$= P(E) + P(F) - P(E \cap F) + P(G) - P(E \cap G) - P(F \cap G) + P(E \cap F \cap G)$$
$$= P(E) + P(F) + P(G) - P(E \cap F) - P(E \cap G) - P(F \cap G) + (E \cap F \cap G)$$

Some Simple Propositions

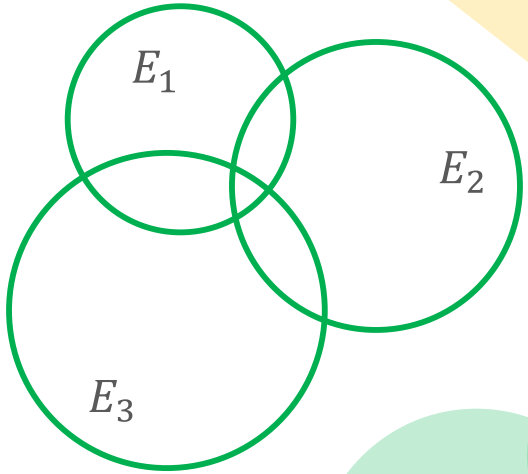
Axiom 1	$0 \le P(E) \le 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

- **Proposition 4**

$$P(E_1 \cup E_2 \cup \cdots \cup E_n)$$
$$= \sum_{i=1}^{n} P(E_i) - \sum_{i_1<i_2} P\left(E_{i_1}E_{i_2}\right) + \cdots + (-1)^{r+1} \sum_{i_1<i_2<\cdots<i_r} P\left(E_{i_1}E_{i_2}\cdots E_{i_r}\right) + \cdots + (-1)^{n+1} P(E_1E_2\cdots E_n)$$

$\sum_{i_1<i_2<\cdots<i_r} P\left(E_{i_1}E_{i_2}\cdots E_{i_r}\right)$ 的組合總數，其實就是$\binom{n}{r}$個組合。

r則是從整個事件的集合$\{1,2,\dots,n\}$的子集合事件數量。



Some Simple Propositions

Axiom 1	$0 \le P(E) \le 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

- **Proposition 4.**

1. 係數項

假設實驗結果會出現在m個E_i事件中，且$m > 0$。那麼這個結果會出現在$\bigcup_i E_i$ 中，其機率會在$P(\bigcup_i E_i)$被計算到一次；同時，它也會在$E_{i_1} E_{i_2} \cdots E_{i_k}$被計算到$\binom{m}{k}$次。

所以它的機率就可以表示為:

$$1 = \binom{m}{1} - \binom{m}{2} + \binom{m}{3} - \cdots \pm \binom{m}{m}$$

Some Simple Propositions

Axiom 1	$0 \leq P(E) \leq 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

$$1 = \binom{m}{1} - \binom{m}{2} + \binom{m}{3} - \cdots \pm \binom{m}{m}$$

因為1可表示成 $\binom{m}{0}$，所以上方的數學式可以轉為:

$$\sum_{i=0}^{m} \binom{m}{i} (-1)^i = 0$$

後方的0可以在拆成$(-1+1)$，此時就可以套用二項式定理得出:

$$0 = (-1+1)^m = \sum_{i=0}^{m} \binom{m}{i} (-1)^i (1)^{m-i}$$

Some Simple Propositions

Axiom 1	$0 \leq P(E) \leq 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

- **Proposition 4.**

2. 簡化 $P(E_1 \cup E_2 \cup \cdots \cup E_n)$

$$= \sum_{i=1}^{n} P(E_i) - \sum_{i_1<i_2} P(E_{i_1}E_{i_2}) + \cdots + (-1)^{r+1} \sum_{i_1<i_2<\cdots<i_r} P(E_{i_1}E_{i_2}\cdots E_{i_r}) + \cdots + (-1)^{n+1} P(E_1E_2\cdots E_n)$$

我們可以觀察到偶數個數的交集都是需要被扣除的，基數個數是需要被加回來的。因此我們就可以簡化整個inclusion-exclusion identity數學式為:

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{r=1}^{n} (-1)^{(r+1)} \sum_{i_1<\cdots<i_r} P(E_{i_1} \ldots E_{i_r})$$

Some Simple Propositions

Axiom 1	$0 \le P(E) \le 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

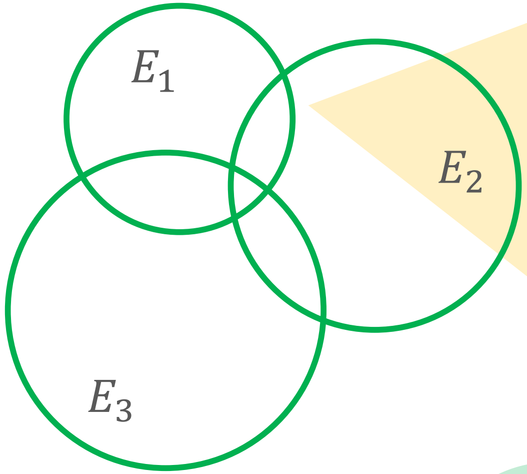
- **Proposition 4.**

3. 從inclusion-exclusion identity中，我們可以得到**聯集的機率上下界範圍**:

$$P\left(\bigcup_{i=1}^{n} E_i\right) \le \sum_{i=1}^{n} P(E_i)$$

$$P\left(\bigcup_{i=1}^{n} E_i\right) \ge \sum_{i=1}^{n} P(E_i) - \sum_{j<i} P(E_iE_j)$$

$$P\left(\bigcup_{i=1}^{n} E_i\right) \le \sum_{i=1}^{n} P(E_i) - \sum_{j<i} P(E_iE_j) + \sum_{k<j<i} P(E_iE_jE_k)$$



Some Simple Propositions

Axiom 1	$0 \leq P(E) \leq 1$
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Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

Proof:

$$P\left(\bigcup_{i=1}^{n} E_i\right) = E_1 \cup E_1^c E_2 \cup E_1^c E_2^c E_3 \cup \cdots \cup E_1^c \ldots E_{n-1}^c E_n$$

$$P\left(\bigcup_{i=1}^{n} E_i\right) = P(E_1) \cup P(E_1^c E_2) \cup P(E_1^c E_2^c E_3) \cup \cdots \cup P(E_1^c \ldots E_{n-1}^c E_n)$$

$$P\left(\bigcup_{i=1}^{n} E_i\right) = P(E_1) + \sum_{i=2}^{n} P(E_1^c \ldots E_{i-1}^c E_i)$$

Some Simple Propositions

Axiom 1	$0 \le P(E) \le 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

$$P\left(\bigcup_{i=1}^{n} E_i\right) = P(E_1) + \sum_{i=2}^{n} P(E_1^c \dots E_{i-1}^c E_i)$$

此時，我們令$B_i = E_1^c \cdots E_{i-1}^c = (\cup_{j<i} E_j)^c$，就可將$P(E_i)$表示為:

$$P(E_i) = P(B_i E_i) + P(B_i^c E_i)$$

其意義為:

$$P(E_i) = P(E_1^c \cdots E_{i-1}^c E_i) + P(E_i \bigcup_{j<i} E_j)$$

$$P(E_1^c \cdots E_{i-1}^c E_i) = P(E_i) - P(\bigcup_{j<i} E_i E_j)$$

Some Simple Propositions

Axiom 1	$0 \leq P(E) \leq 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

$$P\left(\bigcup_{i=1}^{n} E_i\right) = P(E_1) + \sum_{i=2}^{n} P(E_1^c \dots E_{i-1}^c E_i)$$

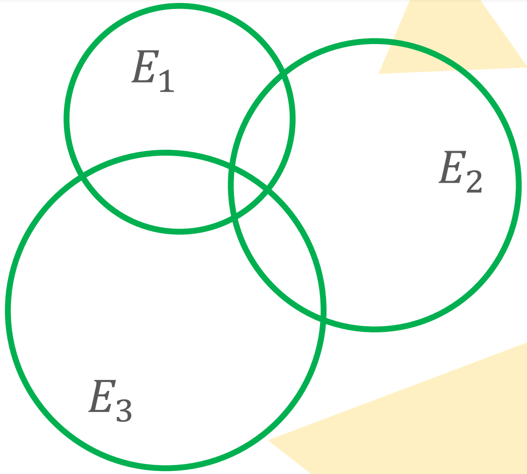
$$P(E_1^c \cdots E_{i-1}^c E_i) = P(E_i) - P(\bigcup_{j<i} E_i E_j)$$

將上式整合可以得出

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i} P(E_i) - \sum_{i} P(\bigcup_{j<i} E_i E_j)$$

Some Simple Propositions

Axiom 1	$0 \leq P(E) \leq 1$
Axiom 2	$P(S) = 1$
Axiom 3	$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$



將上式帶入第一式，可以得出

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i} P(E_i) - \sum_{i} P\left(\bigcup_{j<i} E_iE_j\right)$$

因為機率為非負值(nonnegative)，所以我們可以知道:

$$P\left(\bigcup_{j<i} E_iE_j\right) \leq \sum_{j<i} P(E_iE_j)$$

$$P\left(\bigcup_{j<i} E_iE_j\right) \geq \sum_{j<i} P(E_iE_j) - \sum_{j<i} P(E_iE_jE_iE_k)$$

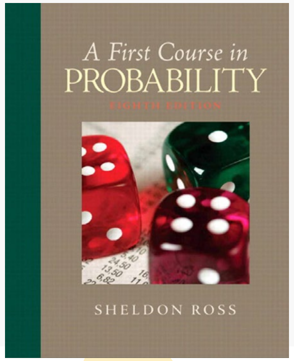
$$P\left(\bigcup_{j<i} E_iE_j\right) \geq \sum_{j<i} P(E_iE_j) - \sum_{j<i} P(E_iE_jE_k)$$

$$P\left(\bigcup_{i=1}^{n} E_i\right) \leq \sum_{i=1}^{n} P(E_i)$$

$$P\left(\bigcup_{i=1}^{n} E_i\right) \geq \sum_{i=1}^{n} P(E_i) - \sum_{j<i} P(E_iE_j)$$

$$P\left(\bigcup_{i=1}^{n} E_i\right) \leq \sum_{i=1}^{n} P(E_i) - \sum_{j<i} P(E_iE_j) + \sum_{k<j<i} P(E_iE_jE_k)$$

[#4] Assignment



- Selected Problems from Sheldon Ross Textbook [1].

1. A box contains 3 marbles: 1 red, 1 green, and 1 blue. Consider an experiment that consists of taking 1 marble from the box and then replacing it in the box and drawing a second marble from the box. Describe the sample space. Repeat when the second marble is drawn without replacing the first marble.

10. Sixty percent of the students at a certain school wear neither a ring nor a necklace. Twenty percent wear a ring and 30 percent wear a necklace. If one of the students is chosen randomly, what is the probability that this student is wearing

(a) a ring or a necklace?
(b) a ring and a necklace?

11. A total of 28 percent of American males smoke cigarettes, 7 percent smoke cigars, and 5 percent smoke both cigars and cigarettes.

(a) What percentage of males smokes neither cigars nor cigarettes?
(b) What percentage smokes cigars but not cigarettes?

12. An elementary school is offering 3 language classes: one in Spanish, one in French, and one in German. The classes are open to any of the 100 students in the school. There are 28 students in the Spanish class, 26 in the French class, and 16 in the German class. There are 12 students that are in both Spanish and French, 4 that are in both Spanish and German, and 6 that are in both French and German. In addition, there are 2 students taking all 3 classes.

(a) If a student is chosen randomly, what is the probability that he or she is not in any of the language classes?
(b) If a student is chosen randomly, what is the probability that he or she is taking exactly one language class?
(c) If 2 students are chosen randomly, what is the probability that at least 1 is taking a language class?

13. A certain town with a population of 100,000 has 3 newspapers: I, II, and III. The proportions of townspeople who read these papers are as follows:

I: 10 percent	I and II: 8 percent	I and II and III: 1 percent
II: 30 percent	I and III: 2 percent	
III: 5 percent	II and III: 4 percent	

(The list tells us, for instance, that 8000 people read newspapers I and II.)

(a) Find the number of people who read only one newspaper.
(b) How many people read at least two newspapers?
(c) If I and III are morning papers and II is an evening paper, how many people read at least one morning paper plus an evening paper?
(d) How many people do not read any newspapers?
(e) How many people read only one morning paper and one evening paper?

[1] Sheldon Ross. A First of Course in Probability. 8th edition.

Reference

Ross, S. (2010). *A first course in probability*. Pearson.

The End

If you have any questions, please do not hesitate to ask me.

Thank you for your attention))